Procedure:

Design

1. Determine $K_r = \dfrac{M_f}{f'_c bd^2}$

2. Calculate ω from the table

3. Determine $\rho = \omega \dfrac{f'_c}{f_y}$

4. Determine $A_s = \rho bd$

Analysis

A_s/bd

1. Determine $\rho = A_p/bd$

2. Determine $\omega = \rho \dfrac{f_y}{f'_c}$

3. Calculate K_r from the table

4. Determine $M_r = K_r f'_c bd^2$

		$\omega_{max.}$										
f_y (MPa)	**f'_c (MPa)**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**65**	**70**	**75**	**80**
	β_1	0.895	0.883	0.870	0.858	0.845	0.833	0.820	0.808	0.795	0.783	0.770
	α_1	0.805	0.798	0.790	0.783	0.775	0.768	0.760	0.753	0.745	0.738	0.730
300	$\omega_{max.}$	0.386	0.377	0.368	0.359	0.351	0.342	0.334	0.325	0.317	0.309	0.301
400	$\omega_{max.}$	0.351	0.342	0.334	0.327	0.319	0.311	0.303	0.296	0.288	0.281	0.274

f'_c	ω	K_r
30 MPa	$0.6600 - \sqrt{0.4355 - 1.553K_r}$	$0.85\omega - 0.6440\omega^2$
35	$0.6531 - \sqrt{0.4266 - 1.5385K_r}$	$0.85\omega - 0.6500\omega^2$
40	$0.6467 - \sqrt{0.4182 - 1.5218K_r}$	$0.85\omega - 0.6571\omega^2$
45	$0.6435 - \sqrt{0.4140 - 1.5142K_r}$	$0.85\omega - 0.6604\omega^2$
50	$0.6345 - \sqrt{0.4025 - 1.4929K_r}$	$0.85\omega - 0.6698\omega^2$
55	$0.6286 - \sqrt{0.3951 - 1.4792K_r}$	$0.85\omega - 0.6760\omega^2$
60	$0.6222 - \sqrt{0.2878 - 1.2623K_r}$	$0.85\omega - 0.6830\omega^2$
65	$0.6164 - \sqrt{0.3799 - 1.4505K_r}$	$0.85\omega - 0.6894\omega^2$
70	$0.6099 - \sqrt{0.3719 - 1.4351K_r}$	$0.85\omega - 0.6968\omega^2$
75	$0.6042 - \sqrt{0.3650 - 1.4216K_r}$	$0.85\omega - 0.7034\omega^2$
80	$0.5976 - \sqrt{0.3571 - 1.4062K_r}$	$0.85\omega - 0.7111\omega^2$

Figure 3.3.4 *Flexural resistance design aid for rectangular section with non-prestressed reinforcement only – precast certified in accordance with CSA A23.4 (ϕ_c =0.70)*

Example 3-4 *Factored flexural resistance of a stadia unit*

Given:

1600 × 800 stadia unit certified in accordance with A23.4

Length = 10.0 m

Span = 9.7 m

Load:

Superimposed dead load: 0.2 kN/m2 = 0.32kN/m

Live Load:

Vertical	4.80 kN/m^2	= 7.68 kN/m
Horizontal	(0.15)(2) kN/m	= 0.30 kN/m
Axial	(0.3)(2)(10) kN	= 6.00 kN

(Axial load acts independently of horizontal load)

Section Properties:

A_g = 298500 mm^2

Weight = 7.0 kN/m

y_b = 306 mm, x_L = 774 mm

I_x = 14500 × 10^6 mm^4

I_y = 72300 × 10^6 mm$_4$

10^6

I_{xy} = $A_g (x - x_L)(y - y_b)$ = 27,720 × 106 mm^4

from tan $2\theta = \dfrac{2I_{xy}}{I_x - I_y}$:

θ = 21.9° = angle of minor axis

I_{minor} = 3350 × 10^6 mm^4

I_{major} = 83,400 × 10^6 mm^4

Concrete:

f'_c = 35 MPa, α_1 = 0.80

β_1 = 0.88 ϕ_c = 0.70

f'_{ci} = 25 MPa

Normal density

Prestressing reinforcement:

9 – 13 mm 1860 MPa low relaxation strands

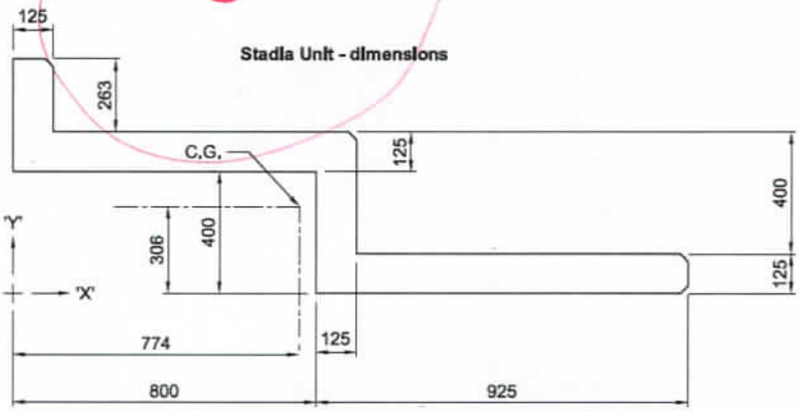
k_p = 0.28

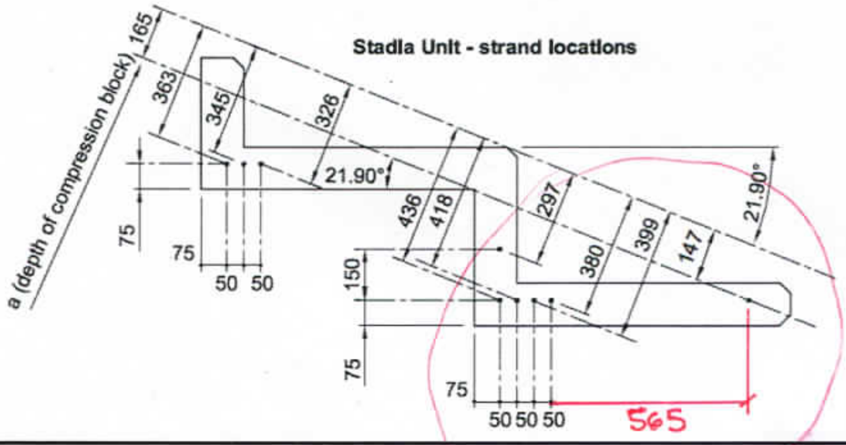
A_p = 99 mm^2/strand

P_o = $0.75A_{p}f_{pu}$ = 1243 kN

P_i = $0.95P_o$ = 1181 kN

P_e = $0.85P_o$ = 1057 kN





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Example 3-4 *Continued*

Problem:

Determine the factored flexural resistance.

Solution:

Due to the shape of the compression block and that failure will occur in bending around the minor axis, use a graphical solution to this problem.

By trial and error, a value of c was found that simultaneously solved the following two equations:

1. $c = \dfrac{\phi_p A_p f_{pr}}{\beta_1 \alpha_1 \phi_c f'_c b}$; and

Stress in prestressing steel at factored resistance:

2. $f_{pr} = f_{pu}(1 - k_p c/d_p)$

$\phi_p = 0.9$

$d_p =$

(363+345+326+297+436+418+380+399+147)/9 = 346 mm

By trial and error:

$$c = 181.3 \text{ mm}$$

$$c/d_p = 181.3/\cancel{370}\ 346 = \cancel{0.49}\ 0.5 \le 0.5$$

$$f_{pr} = 1860[1-(0.28)(\cancel{0.49})\ 0.5] = \cancel{1605}\ 1600 \text{ MPa}$$

Required area of compression block

$$= \frac{(0.9)(9)(99)(\cancel{1605})\ 1600}{(0.80)(0.70)(35)} = 65{,}666 \text{ mm}^2 \quad 65{,}461$$

From the figure above, for the area of the compression block to equal ~~65,666~~ 65,461 mm²:

$$\beta_1 c = \cancel{165}\ 159 \text{ mm}$$

$$c = \cancel{165}\ 159/0.88 = \cancel{181.3}\ 180.7 \text{ mm}$$

The centre of the equivalent compression block is conservatively estimated at $2/3\beta_1 c$ from the top fibre:

$$2/3\ \beta_1 c = (2/3)(0.88)(\cancel{181.3}) = \cancel{106.7} \text{ mm} \quad 106.0$$

$$M_r = \phi_p A_p f_{pr}(d_p - 2/3\beta_1 c)$$

$$= (0.9)(9)(99)(\cancel{1605}\ 1600)(\cancel{370}\ 346 - \cancel{106.7}\ 106.0)/10^6$$

$$= \cancel{338.6} \text{ kN-m} \quad 307.9$$

$$M_f = \frac{(9.7)^2}{8}\{[(7)(1.25)+(0.32)(1.25)$$

$$+(7.68)(1.5)]\cos(21.9^\circ)+(0.3)(1.5)\sin(21.9^\circ)\}$$

$$= 228 \text{ kN-m} < 338.6 \text{ kN-m}$$

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The bending moment around the major axis is small compared to the section depth and can be safely ignored. Reinforcement should be added to resist the axial load.

Procedure:	
Design	**Analysis**
1. Determine $K_r = \dfrac{M_f}{f'_c b d_p^2}$	1. Determine $\rho_p = A_p / b d_p$
2. Calculate ω from the table	2. Determine $\omega_{pu} = \rho_p f_{pu} / f'_c$
3. Determine $\rho_p = \omega_{pu} f'_c / f_{pu}$	3. Calculate K_r from the table
4. Determine $A_p = \rho_p b d_p$	4. Determine $M_r = K_r f'_c b d_p^2$

Figure 3.3.7 *Flexural resistance coefficients for elements with bonded prestressed reinforcement only – precast not certified in accordance with CSA A23.4 ($\phi_c = 0.65$)*

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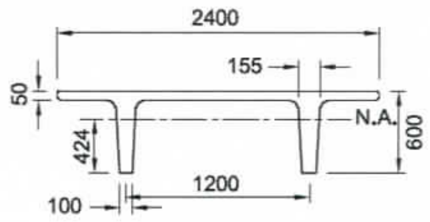
Example 3-7 *Calculation of critical stresses—single point depressed strand*

Given:

2400 x 600 double tee
Span = 21.0 m
Superimposed dead load = 0.5 kN/m^2 = 1.2 kN/m
Superimposed live load = 1.75 kN/m^2 = 4.2 kN/m



Concrete:

f'_c = 35 MPa
f'_{ci} = 25 MPa

Normal Density

Prestressed reinforcement:

14 –13 mm 1860 MPa low-relaxation strands
A_p = (14) (99) = 1386 mm^2
f_{po} = 0.7 f_{pu}

Section properties:

Ag = 260000 mm^2
I = 8580 x 10^6 mm^4
y_b = 424 mm
y_t = 176 mm
S_b = 20200 x 10^3 mm^3
S_t = 48800 x 10^3 mm^3
m = 260 kg/m^2 = 624 kg/m
w = 2.6 kN/m^2 = 6.24 kN/m
e at support = 110 mm
e at midspan = ~~290~~ mm 335
e at 0.4ℓ = ~~254~~ mm 290

Tendon eccentricity as shown:

Problem:

Find critical service load stresses.

Solution:

Prestress force:

P_o = (1386)(0.70)(1860)/10^3 = 1800 kN

P_i = (Assume 10% initial loss)
= (0.90)(1800) = 1620 kN
P_e = (Assume 24% total loss)
= (0.76)(1800) = 1370 kN

Service load moments:

at midspan:

M_{sw} = (6.2)(21)^2 /8 = 344 kN-m
M_{sd} = (1.20)(21)^2 /8 = 66 kN-m
M_ℓ = (4.20)(21)^2 /8 = 232 kN-m

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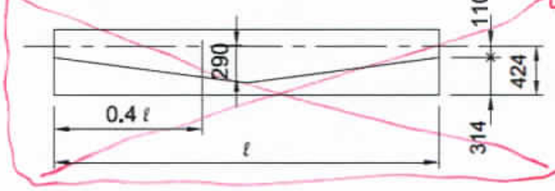

	Support at release P = P_i		Midspan at release P = P_i		0.4 ℓ at service load P = P_e	
	f_b	f_t	f_b	f_t	f_b	f_t
P / A_g	+ 6.2	+ 6.2	+ 6.2	+ 6.2	+ 5.3	+ 5.3
Pe / S	+ 8.8	– 3.7	+ 23.2	– 9.6	+ 17.2	-7.1
M_{sw} / S			– 17.0	+ 7.1	– 16.3	+ 6.7
M_{sd} / S					– 3.1	+ 1.3
M_ℓ / S					– 11.0	+ 4.6
Stresses	+ 15.0	+ 2.5	+ 12.4	+ 3.7	– 7.9	+ 10.8
Allowable stresses	$0.6f'_{ci}$	$0.6f'_{ci}$	$0.6f'_{ci}$	$0.6f'_{ci}$	$0.5\lambda\sqrt{f'_c}$	$0.6f'_c$
	+ 15.0	+ 15.0	+ 15.0	+ 15.0	– 3.0	+ 21.0
	OK	OK	OK	OK	HIGH	OK

SEE ATTACHED PAGE

at 0.4ℓ

M_{sw} = (344)(0.96) = 330 kN-m
M_{sd} = (66)(0.96) = 63 kN-m
M_ℓ = (232)(0.96) = 223 kN-m

Because the extreme fibre stress in the precompressed tensile zone exceeds $0.5\lambda\sqrt{f'_c}$ (~~7.9~~ 5.4 > 3.0), the element is partially prestressed (see Section 3.4.8).

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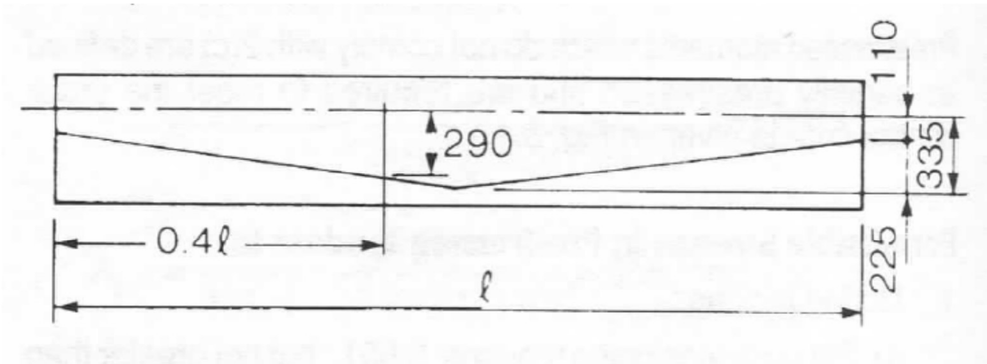

	Support at release $P = P_i$		Midspan at release $P = P_i$		0.4 ℓ at service load $P = P_e$	
	f_b	f_t	f_b	f_t	f_b	f_t
P / A_g	+ 6.2	+ 6.2	+ 6.2	+ 6.2	+ 5.3	+ 5.3
Pe / S	+ 8.8	– 3.7	+ 26.9	– 11.1	+ 19.7	+ 6.8
M_{sw} / S			– 17.0	+ 7.1	– 16.3	– 8.1
M_{sd} / S					– 3.1	+ 1.3
M_ℓ / S					– 11.0	+ 4.6
Stresses	+ 15.0	+ 2.5	+ 16.1	+ 2.2	– 5.4	+ 9.9
Allowable stresses	$0.6f'_{ci}$	$0.6f'_{ci}$	$0.6f'_{ci}$	$0.6f'_{ci}$	$0.5\lambda\sqrt{f'_c}$	$0.6f'_c$
	+ 15.0	+ 15.0	+ 15.0	+ 15.0	– 3.0	+ 21.0
	OK	OK	HIGH	OK	HIGH	OK

$$\frac{M_{cr}}{M_a} = 1 - \left(\frac{f_{t\ell} - f_r}{f_\ell}\right)$$

$f_{t\ell}$ = final calculated total stress in the element

f_ℓ = calculated stress due to live load

A more accurate application of the I_e method is described in a paper by Branson and Trost [8].

***Example 3-16** Deflection calculation using bilinear moment-deflection relationships*

Given:

2400 x 600 double tee of Example 3-7.

Problem:

Determine the total instantaneous deflection caused by the specified uniform live load.

Solution:

$f_r = 0.6\lambda\sqrt{f'_c} = 3.6$ MPa

From Example 3-7, the final tensile stress is 5.4 MPa, which is more than 3.6 MPa, so bilinear behaviour must be considered.

$I_{cr} = n\rho\, bd^3 (1 - k)(1 - k/3)$

$\quad = C\, bd^3$

$A_p = 1386\ mm^2$

d_p at midspan $= e_c + y_t = \cancel{225}\ 335 + 176 = \cancel{401}\ 511$ mm

$$n\rho = \frac{A_p E_p}{E_c b d_p}$$

$$= \frac{(1386)(190000)}{(28200)(2400)(\cancel{401}\ 511)} = \cancel{0.0096}\ 0.0076$$

$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho$

$= \sqrt{(\cancel{0.0096})^2 + 2(\cancel{0.0096})} - \cancel{0.0096}\quad 0.0076$

$= \cancel{0.1292}\ 0.1159$

$C = (0.00\cancel{96}\ 76)(1 - \cancel{0.1292}\ 0.1159)(1 - \cancel{0.1292}\ 0.1159/3)$

$= \cancel{0.0072}\ 0.0065$

$I_{cr} = Cbd_p^3$

$= (\cancel{0.0072}\ 0.0065)(2400)(\cancel{401}\ 511)^3 = \cancel{1114}\ 2082 \times 10^6 mm^4$

Determine the portion of the live load that would result in a bottom tension of 3.6 MPa: at 0.4 L

5.4 – 3.6 = 1.8 MPa

The tension caused by live load alone is 11.0 MPa, therefore, the portion of the live load that would result in a bottom tension of 3.6 MPa is:

$$\left(\frac{11.0 - 1.8}{11.0}\right)(4.2) = 3.5\ kN/m$$

$$\Delta_g = \frac{5w\ell^4}{384 E_c I_g}$$

$$= \frac{(5)(3.5)(21)^4(10^{12})}{(384)(28200)(8580 \times 10^6)} = 37\ mm \downarrow$$

$$\Delta_{cr} = \frac{(5)(0.7)(21)^4(10^{12})}{(384)(28200)(2082 \times 10^6)} = 30\ mm \downarrow$$ ✓

Total instantaneous deflection, $\Delta_\ell = 37 + 30 = 67\ mm \downarrow$

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Example 3-22 *Horizontal shear design for a composite beam*

Given:

Inverted tee beam with 50 mm composite topping, 100 mm over web, see Example 3-5.

Span length = 6.0 m
b_v = 300 mm

Concrete:
Precast: f'_c = 35 MPa
Topping: f'_c = 25 MPa

Prestressed reinforcement:
11 – 13 mm 1860 MPa strands
A_p = (11)(99) = 1089 mm^2

Tie reinforcement:
10M ties (A_v = 200 mm^2)
f_y = 400 MPa

Problem:

Determine the tie requirements to transfer horizontal shear force. Note that this force must include the force in the compression steel which is located in the topping.

$V_{f\ell} = C_1 + C_2 + C4 = 658 + 395 + 204 = 1257$ kN

Solution:

Check maximum interface shear without ties:

~~d = 550 mm~~ $\ell_v = \ell/2 = 6000/2 = 3000$ mm

$V_{r\ell} = 0.7\phi_c b_v \cancel{d}\,\ell_v = (0.7)(0.65)(300)(\cancel{550}\ 3000)/10^3$

$= \cancel{75}\ 410$ kN

This is less than the factored shear, therefore ties are necessary.

Check maximum interface shear with ties:

$V_{r\ell} \le 0.25\,\phi_c f'_c b_v \ell_v$
$= (0.25)(0.65)(25)(300)(3000)/10^3$
$= \cancel{3385}\ 3656$ kN > 1257 kN

The shear force can be resisted by providing sufficient ties across the interface.

Maximum tie spacing:

$s \le 600$ mm

$s \le 4h_f = (4)(100) = 400$ mm

$$s \le \frac{A_v f_y}{0.06\sqrt{f'_c}\,b_v} = \frac{(200)(400)}{(0.06)\sqrt{25}(300)} = 890 \text{ mm}$$

$$v_{f\ell} = \frac{V_{fl}}{b_v \ell_v} = \frac{1257\times10^3}{(300)(3000)} = 1.40 \text{ MPa}$$

$$s \le \frac{(200)(400)(25)}{300}\left[\frac{(0.5)(1.0)(0.65)}{1.40}\right]^2 = 359 \text{ mm}$$

The spacing based on shear friction governs. Use 10M ties @ 350 mm.

Check shear on a vertical plane through the flange at the face of the web.

b_v = 50 mm
$V_{f\ell} = C_1/2 = 658/2 = 329$ kN

Check maximum interface shear with ties:

$V_{r\ell} \le 0.25\phi_c f'_c b_v \ell_v$
$= (0.25)(0.65)(25)(50)(3000)/10^3$
$= 609$ kN > 329 kN

Maximum tie spacing:

$s \le 600$ mm

$s \le 4h_f = (4)(50) = 200$ mm

$$s \le \frac{(100)(400)}{(0.06)\sqrt{25}(50)} = 2670 \text{ mm}$$

$$v_{f\ell} = \frac{V_{fl}}{b_v \ell_v} = \frac{(329)(10^3)}{(50)(3000)} = 2.19 \text{ MPa}$$

$s \le$

$$\frac{(100)(400)(25)}{50}\left[\frac{(0.5)(1.0)(0.65)}{2.19}\right]^2 = 440 \text{ mm}$$

Use 10M bars @ 200 mm governed by 4 times the topping thickness.

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***Example 3-24** Continued*

Problem:
Find the required torsion reinforcement for the spandrel.

Solution:
1. Determine factored loads on spandrel:

D.L. of Beam = (1.25)(10.2)
= 12.75 kN/m

D.L. of deck = (1.25)(4.4)(1.2)(18)/2
= 59.4 kN/stem

L.L. = (1.5)(2.4)(1.2)(18)/2
= 38.9 kN/stem

Concentrated load:

P_f = 59.4 + 38.9 = 98.3 kN/stem

The loads and the resulting values of M_f, V_f and T_f are shown in Fig. 3.7.2.

2. Minimum reinforcement requirements:

Check minimum flexural reinforcement:

$$P_e = A_p f_{pe} = (396)(1030)/10^3 = 408\text{ kN}$$

$$f_{ce} = \frac{P_e}{A_c} + \frac{P_e e}{S_b} = \frac{(408)(10^3)}{435000} + \frac{(408)(677)(10^3)}{170\times10^6} = 2.56\text{ MPa}$$

$$M_{cr} = S_b(0.6\lambda\sqrt{f'_c} + f_{ce}) = \frac{170\times10^6}{10^6}[(0.6)(1.0)\sqrt{35} + 2.56] = 1039\text{ kN-m}$$

$1.2M_{cr}$ = 1246.8 kN-m

4/3 M$_f$ = 896 kN m GOVERNS AT MIDSPAN

Check the flexural resistance:

$$\alpha_1 = 0.85 - (0.0015)(35) = 0.80$$

$$\beta_1 = 0.97 - (0.0025)(35) = 0.88$$

$$\frac{c}{d_p} = \frac{\phi_p A_p f_{pu} + \phi_s A_s f_y - \phi_s A'_s f'_y}{\alpha_1\phi_c f'_c \beta_1 b_w d_p + k_p\phi_p A_p f_{pu}}$$

$$= \frac{(0.9)(396)(1860) + (0.85)(1200)(400) - (0.85)(400)(400)}{(0.8)(0.70)(35)(0.88)(200)(1725) + (0.28)(0.9)(396)(1860)} = 0.152$$

$$f_{pr} = f_{pu}(1 - k_p c/d_p) = (1860)[1-(0.28)(0.152)] = 1781\text{ MPa}$$

$$a = \frac{\phi_p A_p f_{pr} + \phi_s A_s f_y - \phi_s A'_s f'_y}{\alpha_1\phi_c f'_c b_w}$$

$$= \frac{(0.9)(396)(1781) + (0.85)(1200)(400) - (0.85)(400)(400)}{(0.8)(0.70)(35)(200)} = 231\text{ mm}$$

At centreline:

$$M_r = \phi_p A_p f_{pr}\left(d_p - \frac{a}{2}\right) + \phi_s(A_s - A'_s)(f_y)\left(d - \frac{a}{2}\right) + \phi_s A'_s f_y(d - d')$$

$$= 0.9(396)(1781)(1725 - 231/2)\times10^{-6} + 0.85(1200-400)(400)(1830 - 231/2)\times10^{-6} + 0.85(400)(400)(1830-40)\times10^{-6}$$

$$= 1730.9\text{ kN}\cdot\text{m} > \cancel{1.2M_{cr}}$$ 896 kNm

At prestress transfer point (650 mm from end):

$$M_r \approx \frac{1030}{1781}(1021.6) + 466.3 + 243.0$$

$$= 1300.1\text{ kN}\cdot\text{m} > \cancel{1.2M_{cr}}$$ 223 kNm (4/3 M$_f$)

(b) Minimum shear reinforcement:

$$\frac{A_v}{s} = 0.06\sqrt{f'_c}\frac{b_w}{f_y} = 0.06\sqrt{35}\frac{200}{400} = 0.177\text{ mm}^2/\text{mm}$$

Using 10M closed stirrups (A = 100 mm2), the spacing is:

$$s = \frac{(2)(100)}{0.177} = 1130\text{ mm}$$

3. Determine need for torsional reinforcement:

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Continued

Example 3-24 *Continued*

$$T_f = 49.2\ \text{kN-m}$$

$$f_{cp} = \frac{(408)(10^3)}{435\,000} = 0.94\ \text{MPa}$$

$$p_c = 1875 + 200 + 1575 + 200 + 300 + 400 = 4550\ \text{mm}$$

$$T_{cr} = \frac{0.38A_c^2\lambda\phi_c\sqrt{f_c'}}{p_c}\left(\sqrt{1+\frac{\phi_p f_{cp}}{0.38\lambda\phi_c\sqrt{f_c'}}}\right)$$

$$= \left[\frac{(0.38)(435000)^2(1.0)(0.70)\sqrt{35}}{(4550)(10^6)}\right] \times\left[\sqrt{1+\frac{(0.9)(0.94)}{(0.38)(1.0)(0.70)\sqrt{35}}}\right] = 81.2\ \text{kN-m}$$

$0.25\ T_{cr} = 20.3\ \text{kN-m} < T_f$

Torsion reinforcement is required in zone A.

4. Determine section parameters:

$d_v = 0.72\,h = (0.72)(1875) = 1350\ \text{mm}$

or $= 0.9\,d = (0.9)(1725) = 1553\ \text{mm}$

$d_v = 1553\ \text{mm}$

$A_{oh} = (138)[1875-(2)(31)] + (200)(238)$

$= 297000\ \text{mm}^2$

$p_h = (138+1813)(2) + (200)(2) = 4302\ \text{mm}$

$A_o = 0.85A_{oh}$

$= (0.85)(297000) = 252500\ \text{mm}^2$

5. Design shear and torsion reinforcement:

Zone A, at a distance 1600 mm from face of support:

Note: For precast framing, the face of the support is taken to be the centreline of the support.

$V_f = 174.7\ \text{kN}$

$T_f = 29.5\ \text{kN-m}$

$M_f = 439.7\ \text{kN-m}$

$$\varepsilon_x = \frac{\dfrac{M_f}{d_v} + \sqrt{(V_f - V_p)^2 + \left(\dfrac{0.9p_hT_f}{2A_o}\right)^2} - A_pf_{po}}{2(E_pA_p + E_sA_s)}$$

$$= \frac{\left(\dfrac{439.7\times10^6}{1553}\right) + \sqrt{(174.7\text{x}10^3)^2 + \left(\dfrac{0.9(4302)(29.5\times10^6)}{2(252500)}\right)^2 - (396)(0.7)(1860)}}{2((190000)(396)+(200000)(1200))}$$

$= \cancel{0.000902}$ 0.000085

$\theta = 29 + 7000\varepsilon_x$

$= \cancel{35.3°}$ 29.6°

$$s_{ze} = \frac{35S_z}{15+a_g}$$

assume $a_g = 20\ \text{mm}$

$$s_{ze} = \frac{(35)(1553)}{15+20} = 1553\ \text{mm},\ \text{use } s_{ze} = 300\text{mm}$$

$$\beta = \left(\frac{0.4}{1+1500\varepsilon_x}\right)\left(\frac{1300}{1000+s_{ze}}\right)$$

$$= \left(\frac{0.4}{1+(1500)(\cancel{0.000902}\ 0.000085)}\right)\left(\frac{1300}{1000+300}\right)$$

$= \cancel{0.170}$ 0.355

$V_c = \phi_c\lambda\beta\sqrt{f_c'}b_wd_v$

$= (0.7)(1)(\cancel{0.170}\ 0.355)(\sqrt{35})(200)(1553)\text{x}10^{-3}$

$= \cancel{218.7}$ kN 456.6

$= V_c > V_f$

Only minimum stirrups are required.

$$\frac{A_t}{s} = \frac{T_f}{2A_o\phi_sf_y\cot\theta}$$

$$= \frac{(29.5)(\tan \cancel{35.3}\ 29.6°)(10^6)}{(2)(252600)(0.85)(400)}$$

$= 0.\cancel{122}$ mm^2/mm 0.098

Use 10M closed stirrups (A_s = 100 mm²)

s = (100)/($\cancel{0.122}$ 0.098) = $\cancel{820}$ mm 1020

Check need to halve spacing:

$0.125\lambda\phi_cf_c'b_wd_v + V_p$

$= (0.125)(1.0)(0.7)(35)(200)(1553) + 0.0$

$= 951.2\ \text{kN} > V_f$ OK.

$s_{min} = 0.7d_v$

$= (0.7)(1553)$

$= 1087\ \text{mm} > 600\ \text{mm}$

However, $T_f > 0.25\ T_{cr}$, stirrup spacing must be halved to 300 mm.

Use 10M Stirrups at 300 mm O.C.

Continued

OUTSIDE SQUARE ROOT

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Example 3-24 *Continued*

Check web crushing:

$$=\sqrt{\left(\frac{V_f - V_p}{b_w d_v}\right)^2 + \left(\frac{T_f p_h}{1.7A_{oh}{}^2}\right)^2} \le 0.25\phi_c f'_c$$

$$=\sqrt{\left(\frac{(174.7)(10^3)}{(200)(1553)}\right)^2 + \left(\frac{((29.5)(10^6))(4302)}{(1.7)(297000)^2}\right)^2}$$

$$=1.016 \le 0.25(0.7)(35) = 6.1\text{MPa}$$

Calculate actual Vs (based on stirrups provided):

$$V_s = \phi_s\left(\frac{A_v}{S}\right)f_y d_v \cot\theta$$

$$= (0.85)\left(\frac{200}{300}\right)(400)(1553)(\cot \overset{29.6}{\cancel{35.3}}{}^\circ)(10^{-3})$$

$$= \cancel{497}\ \text{kN}$$

620

Vs LIMITED TO V_f* = 174.7 kN

Longitudinal reinforcement:

$$F_t = \frac{M_f}{d_v} + \sqrt{(V_f - 0.5V_s - V_p)^2 + \left(\frac{0.45 p_h T_f}{2A_o}\right)^2}\cot\theta$$

$$= \frac{(439.7)(10^6)}{1553}$$

$$+\sqrt{\begin{aligned}&(174.7(10^3) - 0.5(\overset{174.7}{\cancel{497}})(10^3) - 0)^2\\ &+\left[\frac{(0.45)(4302)(29.5)(10^6)}{2(252500)^{\cancel{2}}}\right]^2\end{aligned}}\ (\cot \overset{29.6}{\cancel{35.3}}{}^\circ)$$

$$= \cancel{(}283129 + \overset{142898}{\cancel{104231}})(10^{-3})\ (\cot \overset{29.6}{\cancel{35.3}}{}^\circ)$$

534.7 $= \cancel{387}$ kN

$$\phi_s A_s f_y + \phi_p A_p f_{pr} = [(0.85)(1200)(400) + (0.9)(396)(1781)] \times 10^{-3}$$

$$= 1043\ \text{kN} > \overset{}{\cancel{593.5}}\ \text{kN}$$

534.7

No additional longitudinal reinforcement is required.

Zone B: (Figure 3.7.2) at 2675 mm from support.

No torsion design is required since $T_f < 0.25\,T_{cr}$.

Check crushing:

$$v_f = \frac{V_f - V_p}{b_w d_v}$$

$$= \frac{(65)(10^3)}{(200)(1600)} = 0.203\ \text{MPa}$$

$$\frac{v_f}{\lambda\phi_c f'_c} = \frac{0.203}{(1.0)(0.70)(35)} = 0.0083 < 0.25\ \text{OK.}$$

$$\varepsilon_x = \frac{\dfrac{M_f}{d_v} + (V_f - V_p) - A_p f_{po}}{2(E_s A_s + E_p A_p)}$$

$$= \frac{\dfrac{602\times10^6}{1553} + \left(65\times10^3 - 0\right) - (396)(0.7)(1860)}{2((200000)(1200) + (190000)(396))}$$

$$= -0.00010 < -0.0002$$

$$\varepsilon_x = 0.0$$

$$\theta = 29 + 7000\varepsilon_x$$

$$= 29.0^\circ$$

$$\beta = \left(\frac{0.40}{1+1500\varepsilon_x}\right)\left(\frac{1300}{1000+s_{ze}}\right)$$

$$= 0.40$$

$$V_c = \lambda\phi_c\beta\sqrt{f'_c}\,b_w d_v$$

$$= (1.0)(0.70)(0.40)\sqrt{35}(200)(1553)/10^3$$

$$= 514\ \text{kN} > V_f$$

Minimum stirrups are required for zone B, the requirements for hanger steel and ledge reinforcement at point loads will govern, so that this type of member will still have transverse reinforcement.

With minimum shear reinforcement:

s = 1130 mm for 10M stirrups.

Use s = 600 mm o.c. $T_f < 0.25\,T_{cr}$

$$V_s = \phi_s \frac{A_v}{s} f_y d_v \cot\theta$$

$$= (0.85)\frac{(200)}{600}(400)(1553)(\cot \overset{29.6}{\cancel{35.3}}{}^\circ)$$

$$= \cancel{248.6}\ \text{kN}$$

~~317.5~~

309.8

Continued

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* CLAUSE 11.3.9.2

Note: Potential torsion due to the end point load must be carefully examined with respect to bearing and end torsion connection locations. Also, shear and torsion caused by loads applied within a distance d_v from the face of the support needs to be considered for bottom loaded beams.

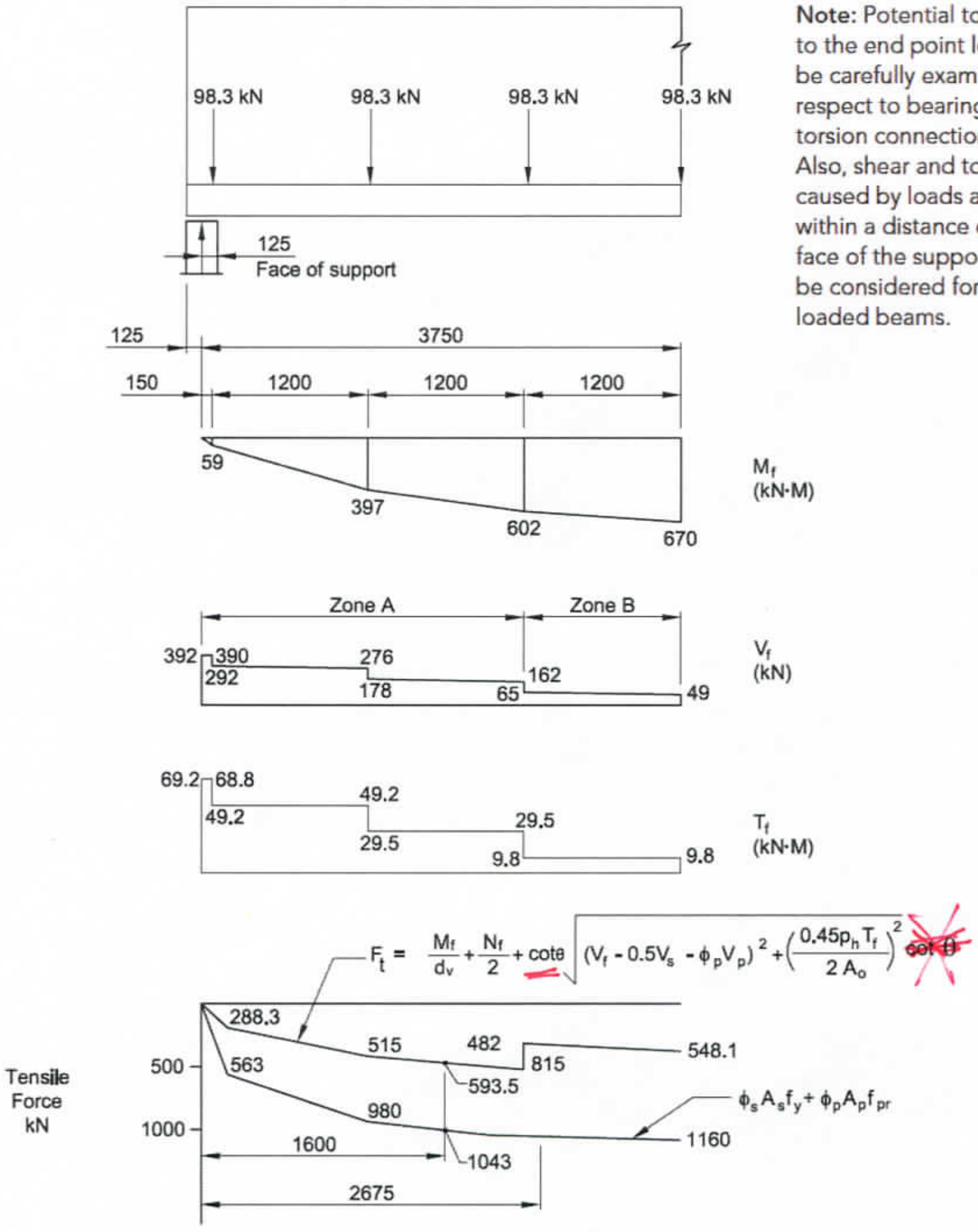

Figure 3.7.2 *Force diagrams for Example 3-24*

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Example 3-31 *Continued*

$f_{tpres} = -6.50$ MPa (at transfer)

$= -6.00$ MPa (at 40 days)

$f_{tself} = +7.00$ MPa for gravity moment:

$M_g = 1950$ kN-m

Problem:

Evaluate the lateral stability of this beam during handling and transporting.

Solution:

(A) Handling for a hanging beam

1) Initial eccentricity offset factor

$= (36000/40000)^2 - 1/3$

$= 0.477$

lateral sweep = 20 mm

lift loop placement offset = 6 mm

$e_i = (20)(0.477) + 6 = 16$ mm

2) Height of roll axis above C.G. of beam

camber = 50 mm

$Y_r = 960 - 50(0.477) = 936$ mm

3) Initial roll angle

$\theta_i = 16 / 936 = 0.0170$ rad

4) Theoretical lateral deflection

$$\bar{Z}_o = \frac{12.8}{(12)(28200)(14900\times10^6)(40000)}[(0.1)(36000)^5 - (2000)^2(36000)^3 + (3)(2000)^4(36000) + (6/5)(2000)^5]$$

$= 372$ mm

5) Tilt angle at cracking

$$f_r = 0.6\sqrt{35} = 3.55\,\text{MPa}$$

$$M_{lat} = \frac{(f_{tpres} + f_{tself} + f_r)I_y}{(b_t/2)}$$

$$= \frac{(-6.50 + 7.00 + 3.55)(14900\times10^6)/10^6}{(900/2)}$$

$= 134$ kN-m

$$\theta_{max} = \frac{M_{lat}}{M_g} = \frac{134}{1950} = 0.0688\text{ rad}$$

6) Tilt angle at failure

$$\theta'_{max} = \sqrt{\frac{16}{(2.5)(372)}} = 0.1312\text{ rad}$$

7) Theoretical lateral deflection at tilt angle

$$\bar{Z}'_o = (372)[1 + (2.5)(0.1312)] = 494\text{ mm}$$

8) Factor of safety against cracking

$$FS = \frac{1}{(372/936 + 0.0170/0.0688)} = 1.55 > 1.0\text{ OK}$$

9) Factor of safety against failure

$$FS' = \frac{(936)(0.1312)}{(494)(0.1312) + 16} = 1.52 < 1.55$$

Therefore, FS′ = FS = 1.55 > 1.5 OK

Note: If the girder was supported 0.5 m from each end then FS = 1.15 and FS′ = 1.15 < 1.5

(B) Transporting when supported from below

1) Radius of stability

rotational stiffness of vehicle = K_O = 4250 kN-m

$$r = \frac{4250\times10^6}{512\times10^3} = 8300\text{ mm}$$

2) Initial eccentricity

placing offset on truck = 25 mm

$e_i = (20)(0.477)+25 = 35$ mm

3) Height of C.G. beam above roll axis

camber = 50 mm

height of C.G. beam above road

= 1800 + 100 + 940 = 2840 mm

height of roll axis above road

= ~~700 mm~~ 600 mm

Y = 2840 + 5(50)(0.477)–600 = 2264 mm

4) Theoretical lateral deflection

$$\bar{Z}_o = \frac{12.8}{(12)(31000)(14900\times10^6)(40000)}[(0.1)(36000)^5 - (2000)^2(36000)^3 + (3)(2000)^4(36000) + (6/5)(2000)^5]$$

$= 338$ mm

5) Tilt angle at cracking

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Continued